The systematic formulation of the discrete maximum principle does, however, provide an easier method of obtaining the necessary derivatives than the rather extensive, complex objective function and its derivatives that Lee evolves.

The weak version of the discrete maximum principle to which the discussion so far applies, is general to all systems. It states that the Hamiltonian function has a stationary value when the objective function has a stationary value.

The strong version, which does not apply generally to all systems, due to the neglect of second order derivatives, states that the stationary points of both the Hamiltonian and the objective functions are similar in nature. The two versions have been discussed fully in (*4*).

The discrete maximum principle has been applied to the same chemical process as Lee used as an example for his Lagrange multiplier approach. The equations and derivatives obtained were precisely Equations (20) to (36) derived in Lee's paper.

It was revealed that the strong principle could be applied. At each stage the Hamiltonian was maximized independently as a function of only one variable, to give a new decision policy. The complete system was re-evaluated using the new decision policy, to generate a further set of values for the state and adjoint variables. The procedure was repeated until changes in the decision policy became insignificant. The vicinity of the optimum was attained in a few iterations.

An advantage of this method is that the number of complete system evaluations is greatly reduced. This is a major saving in time in the complex case.

It also eliminates the difficulties of applying multivariable optimization techniques using derivatives, which often become exceedingly slow when the number of variables is large. Instead the problem is reduced to several single variable searches to maximize the Hamiltonian at each stage.

It must however be stressed that the strong maximum principle cannot be used in every case. It is perhaps unfortunate that Lee has chosen an example in which it is advantageous to use the discrete maximum principle.

NOTATION

f_i^n = the transformation function of state variable i at stage n
H^n = the Hamiltonian function at stage n
M = number of state variables
N = number of stages
P = objective function
S = number of decision variables at each stage
x_i^n = output state variable i from stage n
$\mathbf{x}^{n+1}$ = input state vector to stage n
Z_i^n = adjoint variable associated with state variable i at stage n
i = indicative of variable
n = indicative of stage

Greek Letters

θ^n = decision variable at stage n
λ_i^n = Lagrange multiplier i at stage n
ϕ = modified objective function

LITERATURE CITED

1. Lee, E. S., *AIChE J.*, *15*, 393 (1969).
2. Fan, L. T., and C. S. Wang, "The Discrete Maximum Principle," Wiley, New York (1944).
3. Jackson, R., *Chem. Eng. Sci.*, 19, 253 (1964).
4. Denn, M. M., *Ind. Eng. Chem. Fundamentals,* 4, 240 (1965).
5. Doust, J. S., "Complex Process Optimisation and Parameter Estimation," Ph.D. thesis, Univ. Birmingham, England (1970).

SI Units in Chemical Engineering

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EDITOR'S NOTE:

With the adoption of the International System of units by the *Journal* for all new manuscripts submitted after July 1, 1971, we would like to have our readers familiar with developments in other countries, notably in Britain, where the transition to SI is now underway. Therefore, below is a revised and up-dated version of an article published by Professor Mullin four years ago (*1*). The author, who is probably better known for his publications in the area of crystallization, has been actively engaged in the United Kingdom program for the changeover to the metric system. He is a member of several British and International Standards committees concerned with metrication, and played an active part in the drafting of the British Standard Specification on SI units (*2*).

There is a world-wide trend toward the adoption of a unified system of measurement based on the metric system. The United Kingdom has passed the half-way mark in its 10-year conversion program, and in some sectors of industry the change has already been completed. In the United States of America and Canada there is a growing awareness of the desirability of adopting an internationally rationalized metric system, and the implications of such a change are being studied at the moment.

The metric system we are talking about is the SI, or International System (Système International d'Unités). SI units have been adopted by the International Organisation for Standardization (ISO) and recommended by a large number of national standards organizations. Much has been written about SI units over the past few years, and it is not the present intention to add to that literature. For an account of the historical background and details of specific uses of SI units, readers are referred to several authoritative accounts already published (*2* to *4*).

HOW TO USE SI UNITS

The SI is a rationalized selection of units in the metric system which individually are not new. There are seven basic units (Table 1) and several derived units having special names (Table 2). These derived units are merely for convenience and they can all be expressed, if desired, in terms of the basic units. A few derived units without special names are given in Table 3. Some SI units are of inconvenient size, but the use of multiplying prefixes overcomes this difficulty. A list of the internationally agreed prefixes is given in Table 4.

Great care should be taken in the use of these prefixes. For example, the prefix should always be written immediately adjacent to the units to be qualified, for example, meganewton (MN), kilojoule (kJ), microsecond (μs), and so on. The primary units, on the other hand, should be spaced apart, for example, N s/m^2 or kg/s m^2. Only one prefix can be applied to a given unit at any one time; thus 1,000 kilograms (the "tonne") is 1 megagram (Mg) and not 1 kilo-kilogram.

The symbol m stands for the basic unit metre and the prefix milli, so to avoid confusion it has to be used very carefully in certain circumstances. For example, mN stands for millinewton while m N denotes the metre-newton. However, the convention of spacing the basic units and writing the prefix close to the basic unit is probably not sufficient safeguard in this case, so for the sake of clarity it is better to write the metre as the second unit, that is, newton-metre (N m).

Another important point is that when a multiple of a basic unit is raised to a power, the power applies to the *whole multiple* and not the basic unit alone. Thus 1 km^2 means $1(\text{km})^2 = 10^6$ m^2 and *not* $1\ \text{k}(\text{m})^2 = 10^3$ m^2.

Not all the prefixes in Table 4 will come into common usage. Indeed there is much to be said for the suggestion to confine the choice of prefixes to those powers of 10 which are multiples of ± 3, for example, μ, m, k, etc. There is also some support for the use of strict SI units, that is, m, kg, s, in scientific publications and to write the power of ten in full, for example, 3×10^{-6} m s^{-1} rather than 3 μm s^{-1}.

THE ADVANTAGES OF SI UNITS

Although the SI is simply a development of the metre-kilogram-second (MKS) system, it is superior to MKS because it is a coherent system of units. By this is meant that the product or quotient of unit quantities in SI yield a unit resultant quantity (for example, 1 N $\times$ 1 m = 1 J or 1 kg $\times$ 1 m $\div$ 1 s^2 = 1 N). No numerical factors are involved, such as 4π which crops up in electrical technology if irrational definitions of basic units are used, or g which tends to appear unexpectedly in relationships which employ the gravitational unit of force.

SI units do not eliminate g, but they do relegate it to its proper place, that is, to situations where the force of gravity is actually involved. For example, the SI unit of force is the newton, defined as the force required to impart an acceleration of 1 m/s^2 to a mass of 1 kg. Thus the *weight* of a mass *m* kilogram is a *force* of *mg* newtons, where *g* (m/s^2) is the local value of the acceleration due to gravity. As a matter of fact, a newton is just about the weight of an apple!

LITERATURE CITED

1. Mullin, J. W., *Chem. Eng.(London)* 176, Sept. 1967.
2. *The International System of Units (SI)*, B.S. 3763: 1970, British Standards Institution, London.
3. *IUPAC Manual of Symbols and Terminology* (Ed. M. L. McGlashlan), Butterworths, London, 1969.
4. *SI—The International System of Units,* H. M. Stationery Office, London, 1970.

TABLE 1. THE BASIC SI UNITS*

Quantity	Name of unit	Unit symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

* Temperature difference is commonly expressed in degrees Celsius (°C) instead of kelvins (K) but the unit interval for the Celsius and kelvin scales is the same: 1°C = 1K. The use of 'deg' to denote a temperature interval is not recommended for use in expressing SI units (see Table 5).

TABLE 2. SOME DERIVED SI UNITS HAVING SPECIAL NAMES

Physical quantity	SI unit	Unit symbol
Force	newton	N = kg m/s^2
Work, energy, quantity of heat	joule	J = N m
Power	watt	W = J/s
Pressure	pascal	Pa = N/m^2
Electric charge	coulomb	C = A s
Electrical potential	volt	V = W/A
Electric capacitance	farad	F = A s/V
Electric resistance	ohm	Ω = V/A
Frequency	hertz	Hz = s^{-1}
Magnetic flux	weber	Wb = V s
Magnetic flux density	tesla	T = Wb/m^2
Inductance	henry	H = V s/A
Luminous flux	lumen	lm = cd sr*
Illumination	lux	lx = lm/m^2

* One steradian (sr) is the solid angle which, having its vertex at the centre of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.

TABLE 3. SOME OTHER DERIVED UNITS

Dynamic viscosity	Pa s or N s/m^2
Kinematic viscosity, diffusivity	m^2/s
Surface energy or surface tension	J/m^2 or N/m
*Enthalpy	J/kg
*Entropy, heat capacity	J/kg K
Thermal conductivity	W/m K
Heat transfer coefficient	W/m^2 K
Mass transfer coefficient	m/s
Electric conductivity	A/V m
Magnetic permeability	H/m
Electric field strength	V/m
Magnetic field strength	A/m
Permittivity	F/m
Luminance	cd/m^2
Electric flux density	C/m^2

* May also be expressed in terms of the mole (mol) or kilogram-mole (kmol).

TABLE 4. PREFIXES FOR UNIT MULTIPLES AND SUBMULTIPLES

10^{-18}	atto	a	10^{1}	deca	da
10^{-15}	femto	f	10^{2}	hecto	h
10^{-12}	pico	p	10^{3}	kilo	k
10^{-9}	nano	n	10^{6}	mega	M
10^{-6}	micro	μ	10^{9}	giga	G
10^{-3}	milli	m	10^{12}	tera	T
10^{-2}	centi	c			
10^{-1}	deci	d			

TABLE 5. CONVERSION FACTORS FOR SOME COMMON UNITS
(An asterisk (*) denotes an exact relationship.)

Length	*1 in.	25.4 mm
	*1 ft	0.304 8 m
	*1 yd	0.914 4 m
	1 mile	1.609 3 km
	*1 Å (angstrom)	10^{-10} m
Time	*1 min	60s
	*1 h	3.6 ks
	*1 day	86.4 ks
	1 year	31.5 Ms
Area	*1 in.^{2}	645.16 mm^2
	1 ft^2	0.092 903 m^2
	1 yd^2	0.836 13 m^2
	1 acre	4046.9 m^2
	1 are	100 m^2
	1 mile^2	2.590 km^2
Volume	1 in.^{3}	16.387 cm^3
	1 ft^3	0.028 32 m^3
	1 yd^3	0.764 53 m^3
	1 UK gal	4546.1 cm^3
	1 US gal	3785.4 cm^3
	1 litre	10^{-3} m^3
Mass	1 oz	28.352 g
	*1 lb	0.453 592 37 kg
	1 cwt	50.802 3 kg
	1 ton	1016.06 kg
	1 tonne	1000 kg
Force	1 pdl	0.138 26 N
	1 lbf	4.448 2 N
	1 kgf	9.806 7 N
	1 tonf	9.964 0 kN
	*1 dyn	10^{-5} N
Temperature difference	*1 °F (°R)	5/9 °C (K)
Energy (work, heat)	1 ft lbf	1.355 8 J
	1 ft pdl	0.042 14 J
	*1 cal (internat.)	4.186 8 J
	1 erg	10^{-7}J
	1 Btu	1.055 06 kJ
	1 Chu	1.900 4 kJ
	1 hp h	2.684 5 MJ
	*1 kW h	3.6 MJ
	1 therm	105.51 MJ
	1 thermie	4.185 5 MJ
Calorific value (volumetric)	1 Btu/ft^3	37.259 kJ/m^3
Velocity	1 ft/s	0.304 8 m/s
	1 mile/h	0.447 04 m/s
	1 knot	4.394 3 m/s
Volumetric flow	1 ft^3/s	0.028 316 m^3/s
	1 ft^3/h	7.865 8 cm^3/s
	1 UK gal/h	1.262 8 cm^3/s
	1 US gal/h	1.051 5 cm^3/s
Mass flow	1 lb/h	0.126 00 g/s
	1 ton/h	0.282 24 kg/s
Mass per unit area	1 lb/in.^{2}	703.07 kg/m^2
	1 lb/ft^2	4.882 4 kg/m^2
	1 ton/sq mile	392.30 kg/km^2
Density	1 lb/in.^{3}	27.680 g/cm^3
	1 lb/ft^3	16.019 kg/m^3
	1 lb/UK gal	99.776 kg/m^3
	1 lb/US gal	119.83 kg/m^3
	1 g/cm^3	1000 kg/m^3
Pressure	1 lbf/in.^{2}	6.894 8 kN/m^2
	1 tonf/in.^{2}	15.444 MN/m^2
	1 lbf/ft^2	47.880 N/m^2
	*1 standard atmos.	101.325 kN/m^2
	*1 at (1 kgf/cm^2)	98.066 5 kN/m^2
	*1 bar	10^5 N/m^2
	1 ft water	2.989 1 kN/m^2
	1 in. water	249.09 N/m^2
	1 in. Hg	3.386 4 kN/m^2
	1 mm Hg (1 torr)	133.32 N/m^2
Power (heat flow)	1 hp (British)	745.70 W
	1 hp (metric)	735.50 W
	1 erg/s	10^{-7} W
	1 ft lbf/s	1.355 8 W
	1 Btu/h	0.293 07 W
	1 Chu/h	0.527 54 W
	1 ton of refrigeration	3516.9 W
Moment of inertia	1 lb ft^2	0.042 140 kg m^2
Momentum	1 lb ft/s	0.138 26 kg m/s
Angular momentum	1 lb ft^2/s	0.042 140 kg m^2/s
Viscosity, dynamic	*1 P (poise)	0.1 N s/m^2
	1 lb/ft h	0.413 38 mN s/m^2
	1 lb/ft s	1.488 2 N s/m^2
Viscosity, kinematic	*1 St (stokes)	10^{-4} m^2/s
	1 ft^2/h	0.258 06 cm^2/s
Surface energy (surface tension)	1 erg/cm^2 (1 dyn/cm)	10^{-3} J/m^2 (10^{-3} N/m)
Mass flux density	1 lb/h ft^2	1.356 2 g/s m^2
Heat flux density	1 Btu/h ft^2	3.154 6 W/m^2
	1 Chu/h ft^2	5.678 4 W/m^2
	*1 kcal/h m^2	1.163 W/m^2
Heat transfer coefficient	1 Btu/h ft^2 °F	5.678 3 W/m^2 K
	1 Chu/h ft^2 °C	5.678 W/m^2 K
Specific enthalpy (latent heat, etc.)	*1 Btu/lb	2.326 kJ/kg
Heat capacity (specific heat)	*1 Btu/lb °F	4.186 8 kJ/kg K
Thermal conductivity	1 Btu/h ft °F	1.730 7 W/m K
	1 kcal/h m °C	1.163 W/m K

TABLE 6. SOME COMMON PHYSICAL CONSTANTS

Avagadro's number	$N_A = 6.023 \times 10^{23}$ mol^{-1}
Boltzmann's constant	$k = 1.3805 \times 10^{-23}$ J/K
Planck's constant	$h = 6.626 \times 10^{-34}$ J s
Gravitational acceleration	$g = 9.807$ m/s^2
Universal gas constant	$R = 8.314$ J/mol K
Stefan-Boltzmann constant	$\sigma = 5.6697 \times 10^{-8}$ W/m^2 K^4
Universal constant of gravitation	$G = 6.67 \times 10^{-11}$ N m^2/kg^2
Electronic charge	$e = 1.602 \times 10^{-19}$ C
Electron volt	$eV = 1.602 \times 10^{-19}$ J
Velocity of light in vacuum	$c = 2.99795 \times 10^8$ m/s
Volume of 1 kmol of ideal gas at s.t.p.	= 22.41 m^3
Standard temperature and pressure (s.t.p.)	= 1.013×10^5 N/m^2 and 273.15 K

TABLE 7. APPROXIMATE VALUES OF SOME COMMON PROPERTIES

Density of water	~ 10^3 kg/m^3
Viscosity of water (18°C.)	~ 10^{-3} N s/m^2
Specific heat capacity of water	~ 4.2 kJ/kg K
Thermal conductivity of water	~ 0.6 W/m K
Latent heat of boiling water	~ 2.3 MJ/kg
Density of air at s.t.p.	~ 1.3 kg/m^3
Viscosity of air at s.t.p.	~ 1.7×10^{-5} N s/m^2
Specific heat capacity of air at s.t.p.	~ 1 kJ/kg K
Thermal conductivity of air at s.t.p.	~ 0.024 W/m K